## Solutions

[QI] Let  $f: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$  be f(x,y) = x + y and  $g: \mathbb{R} \to \mathbb{R}$  be g(x) = -x. There are both linear hence continuous.

[2] We identify M2 (IR) with IR4, giving it a topology, then multiplication

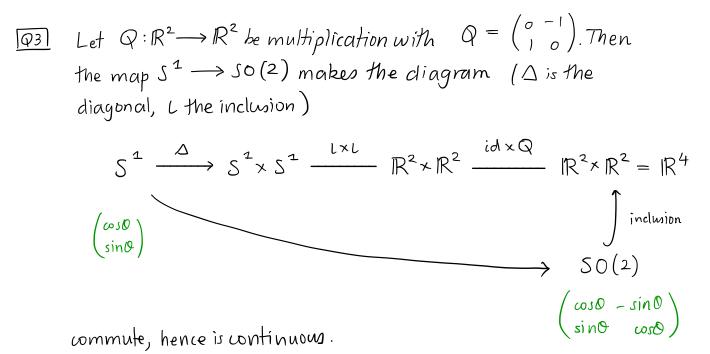
$$\begin{array}{ccc} M_2(\mathbb{R}) \times M_2(\mathbb{R}) & \stackrel{\bullet}{\longrightarrow} & M_2(\mathbb{R}) \\ \mathbb{R}^8 & & \mathbb{R}^4 \end{array}$$

is in each coordinate a polynomial function, hence continuous. This restricts to a continuous map on GL(2, IR), given the subspace topology:

The function  $(-)^{-1}$ :  $GL(2,\mathbb{R}) \longrightarrow GL(2,\mathbb{R})$  is continuous since the composite

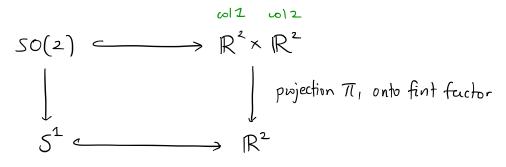
$$\begin{array}{c} GL(2, IR) \longrightarrow GL(2, IR) \longleftrightarrow M_{2}(IR) = IR^{4} \\ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \longmapsto \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \end{array}$$

is writinuous live may check this coordinate-wise, and the only thing to check is that  $\overline{ad-bc}$  is continuous  $U \rightarrow \mathbb{R}$  where  $U \subseteq \mathbb{R}^4$  is where this function is defined, namely  $U = GL(2, \mathbb{R})$ . Hence  $GL(2, \mathbb{R})$  is a topological group.



We already know it is a bij'ection. To show it is a homeomorphism we need to show the inverse is continuous, but that beasy ! The inverse

makes the following diagram commute



and is thus wontinuous.

Q5

Let  $m: G/H \times G/H \longrightarrow G/H$  and  $i: G/H \longrightarrow G/H$  be the multiplication and inverse, and let  $U \subseteq G/H$  be open,  $p: G \longrightarrow G/H$ the quotient. We have to show  $m^{-1}(U)$ ,  $i^{-1}(U)$  are open. Let a point ([9,], [92])  $\in m^{-1}(U) \subseteq G/H \times G/H$  be given. Since

$$G \times G \xrightarrow{m^a} G \xrightarrow{p} G/H$$

is continuous, we can find  $V, W \subseteq G$  with  $g_1 \in V$ ,  $g_2 \in W$  such that

$$(g_{1,g_2}) \in \forall x W \subseteq (\rho \circ m^{\alpha})^{-1}(U),$$
  
1.e.  $\forall a \in V \forall b \in W \quad ab \in \rho^{-1}(U)$ 

Observe that  $m(h, -) : G \longrightarrow G$  is continuous (why?) so

$$\widetilde{V} = (\bigcup_{h \in H} hV) \quad \widetilde{W} = (\bigcup_{h \in H} hW)$$

are open (saturated!) subsets of G, and if  $a \in V$ ,  $b \in W$  and h,  $h' \in H$ then (ha)(h'b) = hh''ab for some  $h'' \in H$ , but this shows that  $(ha)(h'b) \sim ab \sim n$  for some  $a \in U$  so  $(ha)(h'b) \in p^{-1}(U)$  and hence

$$(g_{1},g_{2}) \in \widetilde{V} \times \widetilde{W} \subseteq (p \circ m^{G})^{-1}(U)$$

But then in G/H × G/H

$$([9,],[9_2]) \in \rho(\widetilde{V}) \times \rho(\widetilde{W}) \subseteq m^{(1)}(U)$$

and since  $p(\vec{v}) \times p(\vec{w})$  is open this completes the poorf. The case of  $\vec{z}$  is done similarly.