Minimal models 17 – final $y^3 - x^3$ (checked) ainfmf17

(1)

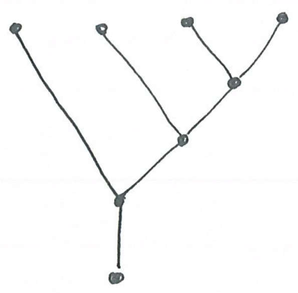
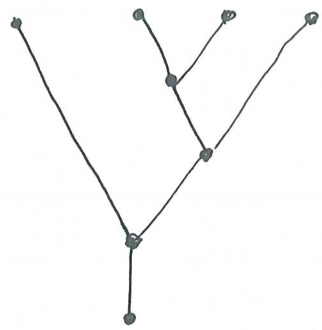
4/12/15

We write down the full minimal model of $W = y^3 - x^3$, using the formula of ainfmf16 p. (11) and the calculations of ainfmf5, ainfmf6, ainfmf10, ainfmf11. What ainfmf16 requires as an input is the amplitude

$$\sum_{\mathcal{C} \in \mathrm{Con}(T)} \mathcal{O}(T, \mathcal{C})(\psi^*_{B_1} \otimes \cdots \otimes \psi^*_{B_q})_{const} \tag{1.1}$$

for each tree T with q inputs and all subsets $B_1, \ldots, B_q \subseteq \{1, \ldots, n\}$. We call this the total amplitude, written $\mathcal{O}(T)(\psi^*_{B_1} \otimes \cdots \otimes \psi^*_{B_q})_{const}$. Since we calculated ρ_3 by hand on p. (19) ainfmf5 we need only write down $q \geq 3$. As explained on p. (4) ainfmf11 in this range only ρ_4, ρ_6 are nonzero.

$\boxed{q = 4}$ As explained in ainfmf6 only

$T_1 =$  $T_2 =$ 

contribute. Regarding the possible $B_j \subseteq \{1, 2\}$ we have to keep in mind the following constraints, discussed in ainfmf11.

⌈signs may be off⌋

- No consecutive H zones (i.e. all $B_j \neq \phi$)

- The total number of ψ_1's (resp. ψ_2's) in the input must be divisible by 3, and is at most 4, so it must be 3.

This means there are 12 possible inputs. Since there is a $\psi_1 \longleftrightarrow \psi_2$ invariance for this potential and $q=4$, there are only six inputs to worry about. Their amplitudes from ainfmf10 are, writing

$$(1\ 2)(1\ 2)(1)(2) \quad \text{for} \quad \psi_1^*\psi_2^* \otimes \psi_1^*\psi_2^* \otimes \psi_1^* \otimes \psi_2^*$$

given by the table

$\mathcal{O}(T)(\psi^*_{B_1} \otimes \cdots \otimes \psi^*_{B_4})_{const}$	T_1	T_2
B_1 B_2 B_3 B_4 $(1\ 2)(1\ 2)(1)(2)$	0	$1/2$
$(1\ 2)(1)(1\ 2)(2)$	$-1/2$	$-1/2$
$(1)(1\ 2)(1\ 2)(2)$	-1	0
$(1\ 2)(1)(2)(1\ 2)$	$1/2$	0
$(1)(1\ 2)(2)(1\ 2)$	1	0
$(1)(2)(1\ 2)(1\ 2)$	$1/2$	0

(2.1)

Now we want to write a formula for $\rho_4(\Lambda_1\otimes\cdots\otimes\Lambda_4)$, using (11.1) of ainfmf16. Now $q=4$ so

$$1+\binom{q}{2}+\sum_i i|\Lambda_i| \equiv 1+\sum_i i|\Lambda_i| \qquad (3.1)$$

For $T=\hat{T}_1$ we have ($T=$ )

$$\sum_{j\geq 1}\binom{M_j}{2}+\sum_i P_i = 1+1+1 \equiv 1 \qquad (3.2)$$

whereas for $T=\hat{T}_2$ ($T=$ )

$$\sum_{j\geq 1}\binom{M_j}{2}+\sum_i P_i = 0+1+2+1 \equiv 0 \qquad (3.3)$$

The subsets $\underline{A}=(A_1,\ldots,A_4)$ with a nonzero contribution are the ones encoded in (2.1) plus those with $1\longleftrightarrow 2$ swapped, so a total of twelve. The sum for ρ_4 can therebefore be written as a sum of operators

$$(\psi_{A_1}\lrcorner\Lambda_1)\cdots(\psi_{A_4}\lrcorner\Lambda_4)+(\psi_{A_1^+}\lrcorner\Lambda_1)\cdots(\psi_{A_4^+}\lrcorner\Lambda_4)$$

where A^+ means swap $1,2$ (as a subset), i.e. $(-)^+:\{1,2\}\to\{1,2\}$ is $1^+=2$, $2^+=1$.

We compute the coefficient of (1.1) of ainfmf16

$$(\psi_{A_1} \lrcorner \Lambda_1) \cdot \cdots \cdot (\psi_{A_q} \lrcorner \Lambda_q) \qquad (4.1)$$

in a table, for $q=4$, so

$$\rho_4(\Lambda_1 \otimes \cdots \otimes \Lambda_4) = (-1)^{1+\sum_i i|\Lambda_i|} \sum_T (-1)^{\sum_j \binom{M_j}{2} + \sum_i P_i}$$

$$\sum_{\underline{A}} (\text{table entry in row } \underline{A} \text{ col } T)(\psi_{A_1} \lrcorner \Lambda_1)\cdots(\psi_{A_q} \lrcorner \Lambda_q)$$

$A_1 \; A_2 \; A_3 \; A_4$		
(2)(1)(1 2)(1 2)	0	$\frac{1}{2}(-1)^{1+1+\|\Lambda_1\|}$
(2)(1 2)(1)(1 2)	$-\frac{1}{2}(-1)^{1+\|\Lambda_1\|+\|\Lambda_2\|}$	$-\frac{1}{2}(-1)^{\|\Lambda_1\|+\|\Lambda_2\|}$
(2)(1 2)(1 2)(1)	$-1\cdot(-1)^{1+1+\|\Lambda_1\|+\|\Lambda_2\|+\|\Lambda_3\|}$	0
(1 2)(2)(1)(1 2)	$\frac{1}{2}(-1)^{1+\|\Lambda_2\|}$	0
(1 2)(2)(1 2)(1)	$1\cdot(-1)^{\|\Lambda_2\|+\|\Lambda_3\|}$	0
(1 2)(1 2)(2)(1)	$\frac{1}{2}(-1)^{1+\|\Lambda_3\|}$	0

Hence

$$\rho_4(\Lambda_1 \otimes \cdots \otimes \Lambda_4) = (-1)^{1+\sum_i i|\Lambda_i|} \cdot \qquad (5.1)$$

$$
\begin{aligned}
&- \Big[\tfrac{1}{2}(-1)^{|\Lambda_1|+|\Lambda_2|} \Big\{ [\psi_2, \Lambda_1] \cdot [\psi_2, [\psi_1, \Lambda_2]] \cdot [\psi_1, \Lambda_3] \cdot [\psi_2, [\psi_1, \Lambda_4]] \\
&\qquad + [\psi_1, \Lambda_1] \cdot [\psi_2, [\psi_1, \Lambda_2]] \cdot [\psi_2, \Lambda_3] \cdot [\psi_2, [\psi_1, \Lambda_4]] \Big\} \\
&\quad - (-1)^{|\Lambda_1|+|\Lambda_2|+|\Lambda_3|} \Big\{ [\psi_2, \Lambda_1] \cdot [\psi_2, [\psi_1, \Lambda_2]] \cdot [\psi_2, [\psi_1, \Lambda_3]] \cdot [\psi_1, \Lambda_4] \\
&\qquad + [\psi_1, \Lambda_1] \cdot [\psi_2, [\psi_1, \Lambda_2]] \cdot [\psi_2, [\psi_1, \Lambda_3]] \cdot [\psi_2, \Lambda_4] \Big\} \\
&\quad - \tfrac{1}{2}(-1)^{|\Lambda_2|} \Big\{ [\psi_2, [\psi_1, \Lambda_1]] \cdot [\psi_2, \Lambda_2] \cdot [\psi_1, \Lambda_3] \cdot [\psi_2, [\psi_1, \Lambda_4]] \\
&\qquad + [\psi_2, [\psi_1, \Lambda_1]] \cdot [\psi_1, \Lambda_2] \cdot [\psi_2, \Lambda_3] \cdot [\psi_2, [\psi_1, \Lambda_4]] \Big\} \\
&\quad + (-1)^{|\Lambda_2|+|\Lambda_3|} \Big\{ [\psi_2, [\psi_1, \Lambda_1]] \cdot [\psi_2, \Lambda_2] \cdot [\psi_2, [\psi_1, \Lambda_3]] \cdot [\psi_1, \Lambda_4] \\
&\qquad + [\psi_2, [\psi_1, \Lambda_1]] \cdot [\psi_1, \Lambda_2] \cdot [\psi_2, [\psi_1, \Lambda_3]] \cdot [\psi_2, \Lambda_4] \Big\} \\
&\quad - \tfrac{1}{2}(-1)^{|\Lambda_3|} \Big\{ [\psi_2, [\psi_1, \Lambda_1]] \cdot [\psi_2, [\psi_1, \Lambda_2]] \cdot [\psi_2, \Lambda_3] \cdot [\psi_1, \Lambda_4] \\
&\qquad + [\psi_2, [\psi_1, \Lambda_1]] \cdot [\psi_2, [\psi_1, \Lambda_2]] \cdot [\psi_1, \Lambda_3] \cdot [\psi_2, \Lambda_4] \Big\} \Big] \\
&+ \Big[\tfrac{1}{2}(-1)^{|\Lambda_1|} \Big\{ [\psi_2, \Lambda_1] \cdot [\psi_1, \Lambda_2] \cdot [\psi_2, [\psi_1, \Lambda_3]] \cdot [\psi_2, [\psi_1, \Lambda_4]] \\
&\qquad + [\psi_1, \Lambda_1] \cdot [\psi_2, \Lambda_2] \cdot [\psi_2, [\psi_1, \Lambda_3]] \cdot [\psi_2, [\psi_1, \Lambda_4]] \Big\} \\
&\quad - \tfrac{1}{2}(-1)^{|\Lambda_1|+|\Lambda_2|} \Big\{ [\psi_2, \Lambda_1] \cdot [\psi_2, [\psi_1, \Lambda_2]] \cdot [\psi_1, \Lambda_3] \cdot [\psi_2, [\psi_1, \Lambda_4]] \\
&\qquad + [\psi_1, \Lambda_1] \cdot [\psi_2, [\psi_1, \Lambda_2]] \cdot [\psi_2, \Lambda_3] \cdot [\psi_2, [\psi_1, \Lambda_4]] \Big\} \Big]
\end{aligned}
$$

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5.5

$$= (-1)^{1+\sum_i i|\Lambda_i|} \Big[$$

$$-(-1)^{|\Lambda_1|+|\Lambda_2|} (2)(1\ 2)(1)(1\ 2)$$

$$-(-1)^{|\Lambda_2|+|\Lambda_3|} (1\ 2)(1)(1\ 2)(2)$$

cyclic

$$-(-1)^{|\Lambda_1|+|\Lambda_2|} (1)(1\ 2)(2)(1 2)$$

$$-(-1)^{|\Lambda_2|+|\Lambda_3|} (1\ 2)(2)(1 2)(1)$$

$$+(-1)^{|\Lambda_1|+|\Lambda_2|+|\Lambda_3|} (2)(1\ 2)(1\ 2)(1)$$

$$+\tfrac{1}{2}(-1)^{|\Lambda_3|} \cdot (1\ 2)(1\ 2)(1)(2)$$

cyclic

$$+\tfrac{1}{2}(-1)^{|\Lambda_2|} \cdot (1\ 2)(1)(2)(1\ 2)$$

$$+\tfrac{1}{2}(-1)^{|\Lambda_1|} (1)(2)(1\ 2)(1\ 2)$$

$$+(-1)^{|\Lambda_1|+|\Lambda_2|+|\Lambda_3|} (1)(1\ 2)(1\ 2)(2)$$

$$+\tfrac{1}{2}(-1)^{|\Lambda_3|} \cdot (1\ 2)(1\ 2)(2)(1)$$

cyclic

$$+\tfrac{1}{2}(-1)^{|\Lambda_2|} (1\ 2)(2)(1)(1\ 2)$$

$$+\tfrac{1}{2}(-1)^{|\Lambda_1|} (2)(1)(1\ 2)(1\ 2) \Big]$$

$\boxed{q=6}$ The calculations begin on p. 4.5 ainfmf11.

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6

The only sets that are relevant as inputs are

$(12)\cdots(12)$ i.e. all $A_i = \{1,2\}$.

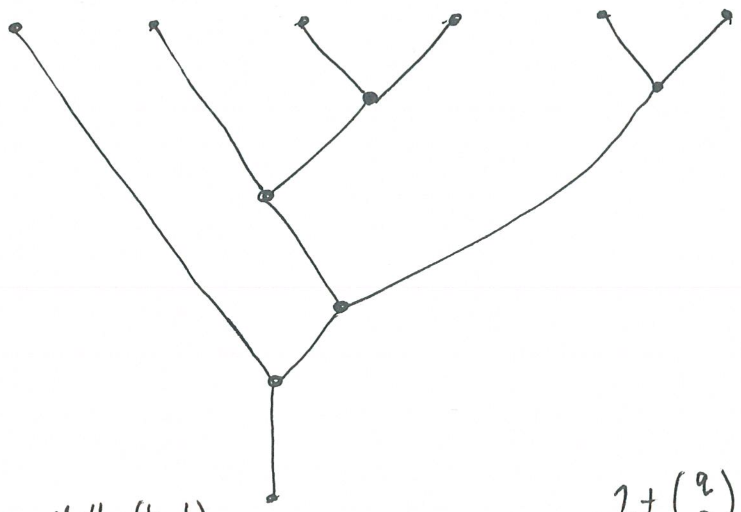
Hence ainfmf16 (11.1) reads (6.1)

$$\rho_6(\Lambda_1 \otimes \cdots \otimes \Lambda_6) = (-1)^{1+\binom{q}{2}+\sum_i i|\Lambda_i|} \sum_T (-1)^{\sum_{j\geq 1}\binom{M_j}{2}+\sum_i P_i}$$

$$\sum_{\mathcal{C}\in \operatorname{Con}(\hat{T})} \mathcal{O}(\hat{T},\mathcal{C})(\psi_1^*\psi_2^* \otimes \cdots \otimes \psi_1^*\psi_2^*)_{const}$$

$$\cdot(\psi_{\{1,2\}} \lrcorner \Lambda_1)\cdots\cdots(\psi_{\{1,2\}} \lrcorner \Lambda_q)$$

In ainfmf11 we compute that the following trees contribute (we draw $\hat{T}$ to match ainfmf11)

$\hat{T}_1 =$  (6.2)

(recall the flip!)

$$1+\binom{q}{2} = 1+\frac{6!}{2!4!}$$
$$= 1+\frac{6\cdot5\cdot4\cdot3\cdot2}{2\cdot4\cdot3\cdot2} = 1+3\cdot5$$
$$= 16 \equiv 0$$

$M_1 = 1$ $P_1 = 0$ (rightmost input in (6.2))

$M_2 = 1$ $P_2 = 1$

$M_3 = 2$ $P_3 = 1$ $P_5 = 2$

$M_4 = 1$ $P_4 = 2$ $P_6 = 1$

$$\sum_j \binom{M_j}{2} + \sum_i P_i \equiv 1+7 \equiv 0.$$

$$\theta(\hat{T}_1)(\psi_1^*\psi_2^* \otimes \cdots \otimes \psi_1^*\psi_2^*)$$

$$= 1/4 \qquad \text{(p. 11, 12 ainfmf11)}$$

There are contributions from the same tree from p. 12 – 13 but they cancel.

No other trees contribute, so

$$\rho_6(\Lambda_1 \otimes \cdots \otimes \Lambda_6) = \frac{1}{4}(-1)^{|\Lambda_1|+|\Lambda_3|+|\Lambda_5|} \tag{7.1}$$

$$[\psi_2,[\psi_1,\Lambda_1]]\cdot[\psi_2,[\psi_1,\Lambda_2]]\cdot[\psi_2,[\psi_1,\Lambda_3]]$$

$$\cdot[\psi_2,[\psi_1,\Lambda_4]]\cdot[\psi_2,[\psi_1,\Lambda_5]]\cdot[\psi_2,[\psi_1,\Lambda_6]]$$