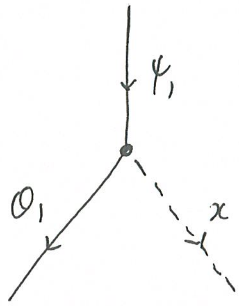
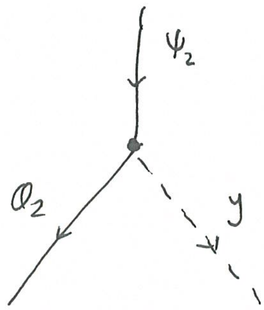
# Minimal models for MFs II (checked)



We continue the calculations for $W = y^3 - x^3$ begun in ainfmf10. Recall the Feynman rules:

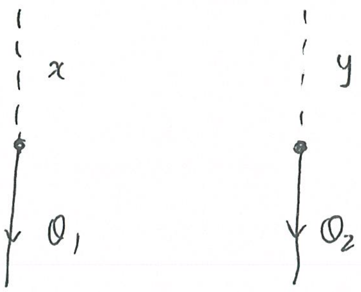



$\boxed{1}$




$\boxed{-1}$ (1.1)

(occur at input or internal edge)




precisely one per $H_\infty$ zone. (occurs at internal edge only) (1.2)


each input from a different leg (occurs at a junction of the tree) (1.3)

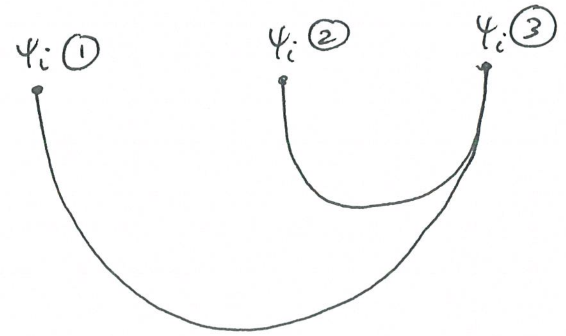
On p. 18 of ainfmf10 we collated the nonzero values of $b_4$ (with vacuum boundary conditions) and in the process of arriving at these values we made various general observations. Our aim in this note is to collect these "general rules" (for $W = y^3 - x^3$) and use them to derive the values of $b_4$ in a more efficient way.

# General rules

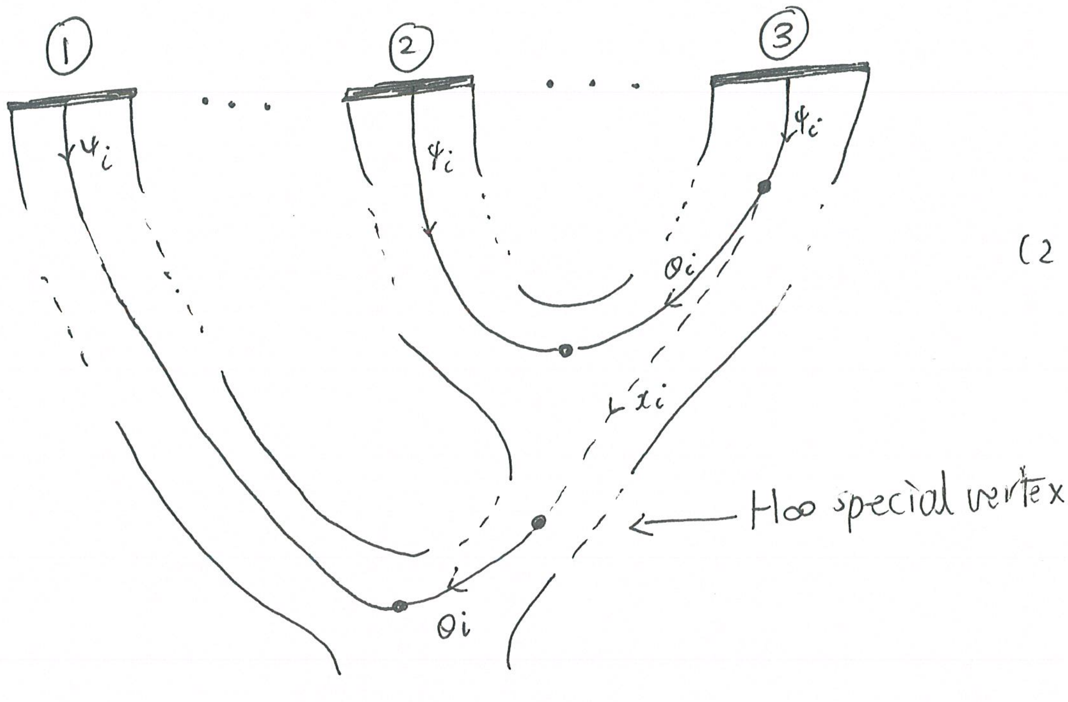
(I) Since $H^2 = 0$ a configuration where two $H_{\infty}$ zones, or a $H_{\infty}$ zone and a $\delta_{\infty}$ zone, are directly connected with no intermediate $\Xi$ vertices, contributes zero to the overall amplitude.

Note This arises via cancellations (e.g. p. 11, 21)

(II) From the Feynman rules we deduce that in a diagram with nonzero amplitude the number of incoming $\varphi_1$'s, $\varphi_2$'s must (separately) be divisible by three, and a given diagram will divide the $\varphi_1$'s, (resp. $\varphi_2$'s) into subsets of size three,




(2.1)




(2.2)

The point is that ②, ③ are connected to ① via $\Xi$-vertices, which select a particular junction  of the tree. To make a nonzero contribution, the diagram must have its $H_\infty$ special vertex on the path in the tree between these two selected junctions (as shown in (2.2)). Otherwise we get zero as shown in ainfmf10 (15.1).

Def$^N$ We call the junctions defined in the previous paragraph the $\Xi$-junctions of the triple, in the particular diagram.

Def$^N$ A special vertex is an interaction of type


Def$^N$ The width of a triple as in (2.2) is the number of edges on the unique path between its $\Xi$-junctions.

Note that the two $\Xi$-junctions are uniquely determined by where the $\psi_i$'s are fed in from (their input channel). The $\Xi$ interaction linking ②, ③ occurs at the point the paths ② $\to$ root and ③ $\to$ root merge (it cannot occur later as then the $\psi_i$, $Q_i$ would be on the same leg). The other $\Xi$-interaction occurs when these two paths further merge with ① $\to$ root.

To compute $b_q$, $q \geq 3$ we have $q$ inputs and $q-2$ internal edges. Each special vertex must be used, and there are $q-2$ of them, so if we define

$$\alpha := (\text{number of } \psi_1 \text{ inputs})/3$$

$$\beta := (\text{number of } \psi_2 \text{ inputs})/3$$

We have that $\alpha + \beta$ is the number of triples, each of which is matched to a special vertex, so

$$\alpha + \beta = q - 2 \qquad (4.1)$$

But on the other hand we have at most one $\psi_1$ or $\psi_2$ per channel (as $\psi_i^2 = 0$) so

$$3\alpha \leq q \quad , \quad 3\beta \leq q \qquad (4.2)$$

Combining gives

$$3(q-2) = 3\alpha + 3\beta \leq 2q$$

$$\therefore \quad q - 6 \leq 0 \Rightarrow q \leq 6.$$

Hence $b_q = 0$ for $q > 6$. For $b_5$ we have $\alpha = 1, \beta = 1$ by (4.2) but with only two triples we only use two special vertices, so (4.2) fails and also $b_5 = 0$. So for $W = y^3 - x^3$ the only nonzero multiplications are

$$b_2, b_3, b_4, b_6. \qquad (4.3).$$

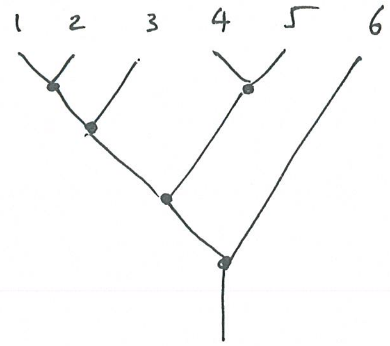
Now $b_2$ is on p. 5 of ainfmf5, $b_3$ is (19.1) there, $b_4$ is (modulo some signs for $\Lambda_0, \ldots, \Lambda_3$) p. 18 of ainfmf10.

Now we turn to $b_6$. There are four special vertices and hence 12 input fermions. So the only input on which $b_6$ is nonzero is

$$\underbrace{\psi_1^*\psi_2^* \otimes \cdots \otimes \psi_1^*\psi_2^*}_{6 \text{ copies}} \qquad (4.5.1).$$

We can describe a tree via bracketing, i.e.

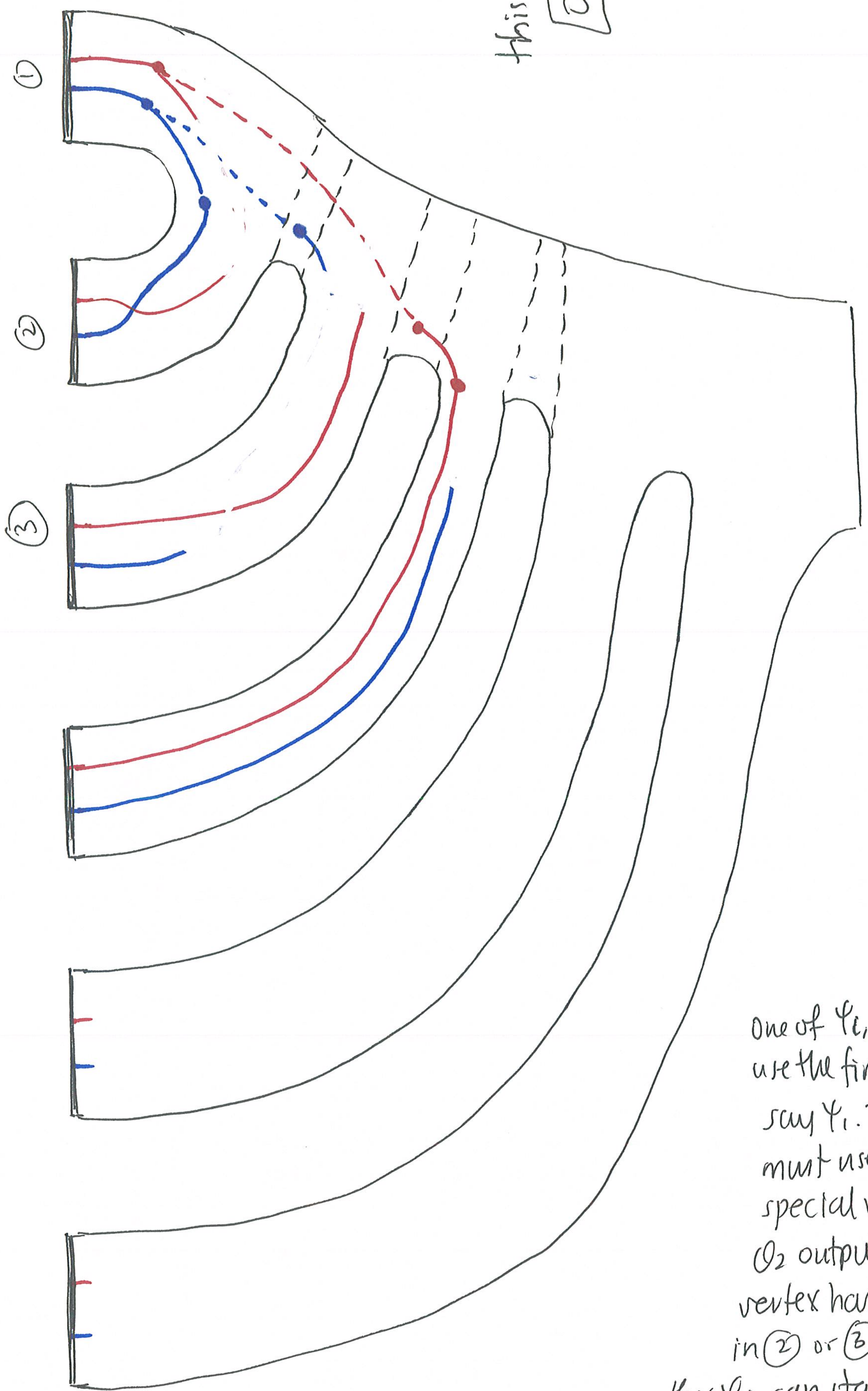
$$((((1\ 2)\ 3)\ (4\ 5))\ 6)$$




We say in this case $(1,2)$ and $(4,5)$ pair "first". We first calculate some examples and then begin a systematic study. Observe that both input in channel 6 are <u>primary</u> (i.e. inputs to trivalent vertices), and we indicate this with $\downarrow$. No input in channel 1 or 2 can be primary, so the possible "first pairs" are

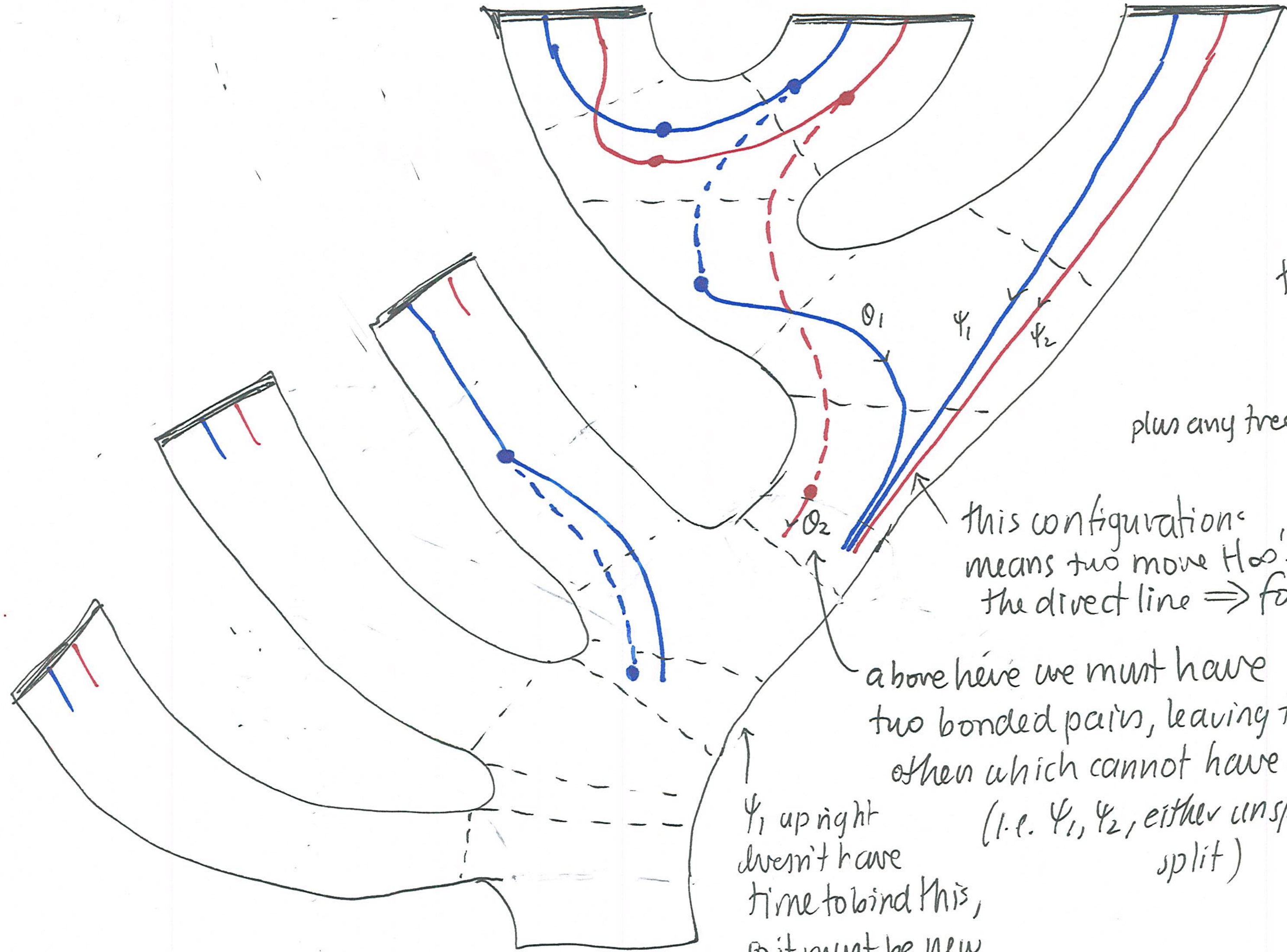
$$(2,3),\ (3,4),\ (4,5),\ (5,6).$$

this tree $\boxed{D}$




(5.1)

One of $\varphi_1, \varphi_2$ needs to use the first special vertex, say $\varphi_1$. Then the other must use the second special vertex, and the $O_2$ output of the trivalent vertex has to be stashed in 2 or 3. But then no other $\varphi_i$ can stash its own $O_i$ in time to use the third special vertex.




this tree.

$\boxed{\theta}$ ...

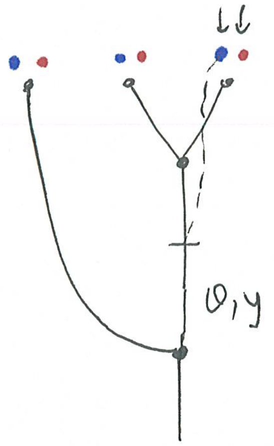
plus any tree with 

this configuration: means two more $H_\infty$'s on the direct line $\Rightarrow$ forces topology.

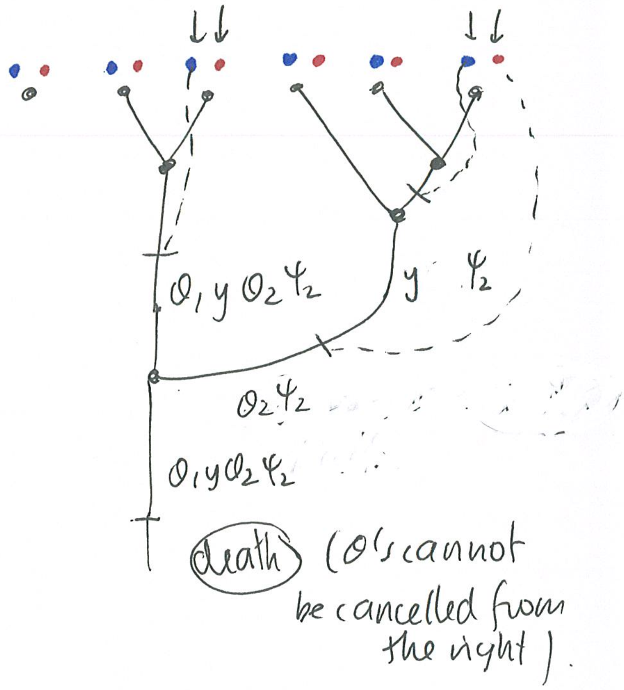
above here we must have two bonded pairs, leaving two others which cannot have interacted (i.e. $\psi_1, \psi_2$, either unsplit or split)

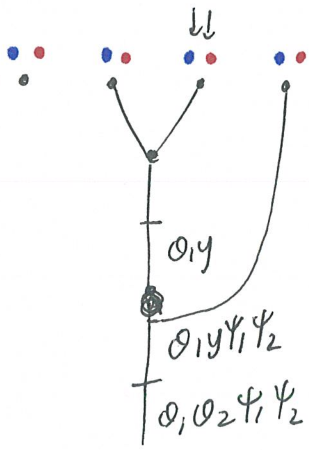
$\psi_1$ upright doesn't have time to bind this, so it must be new $\psi_1$.. But then we get two $\theta_1$'s.

We enumerate the diagrams with $(2,3)$ as a first pair

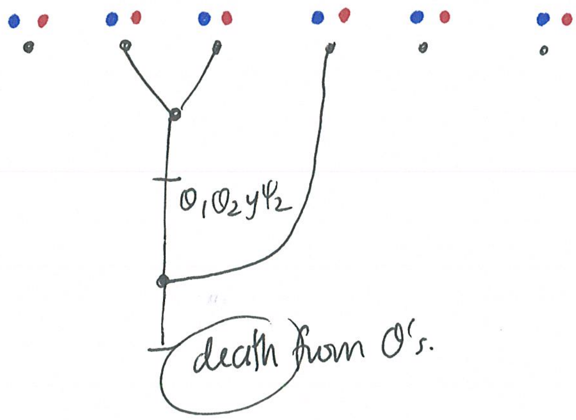


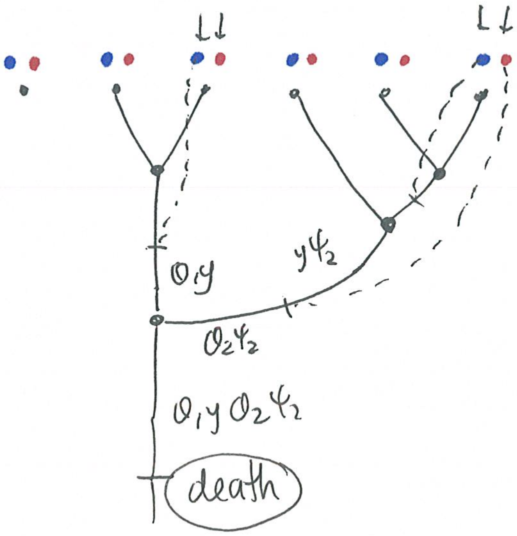

$(7.1)$

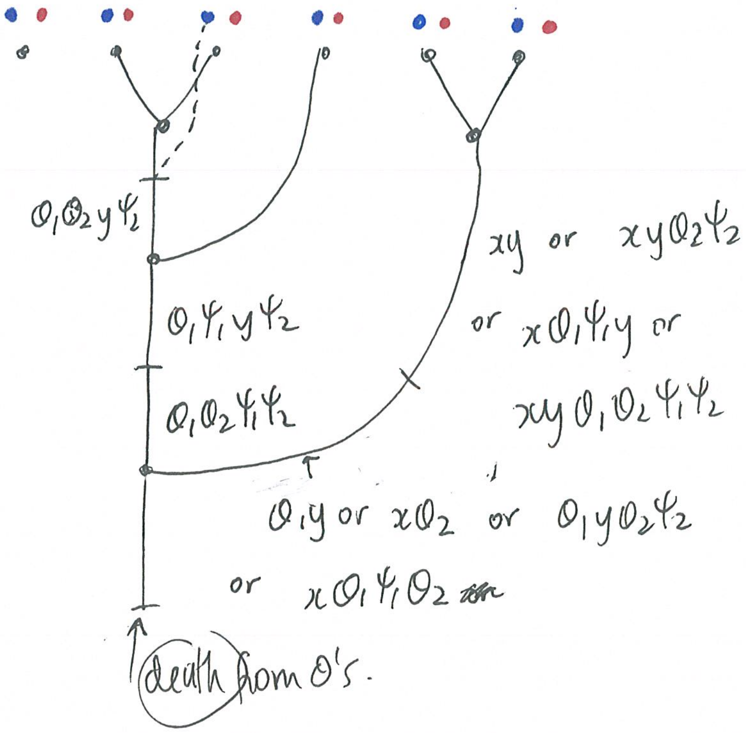


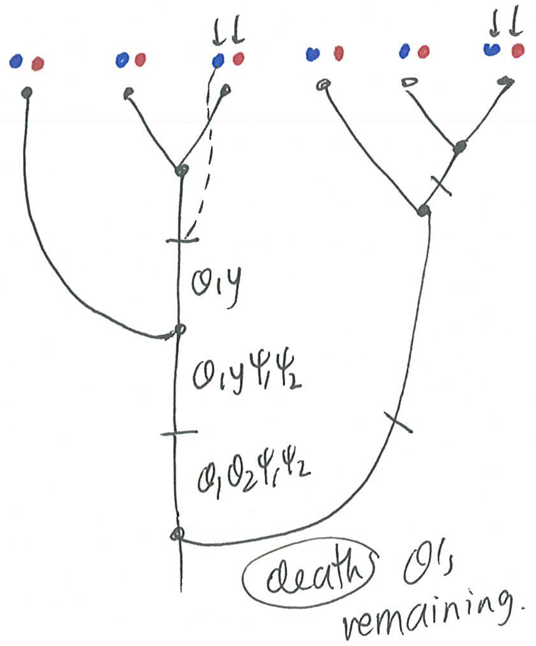

$(7.2)$

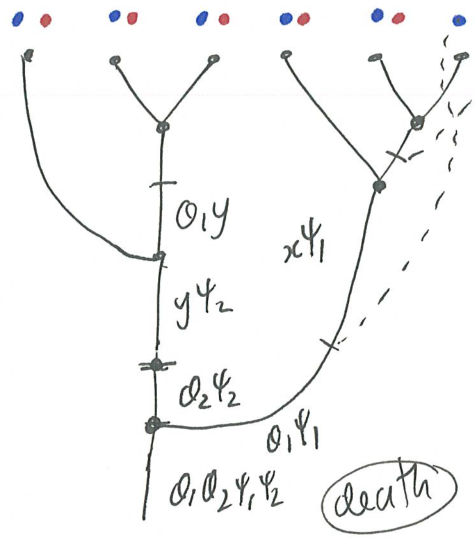


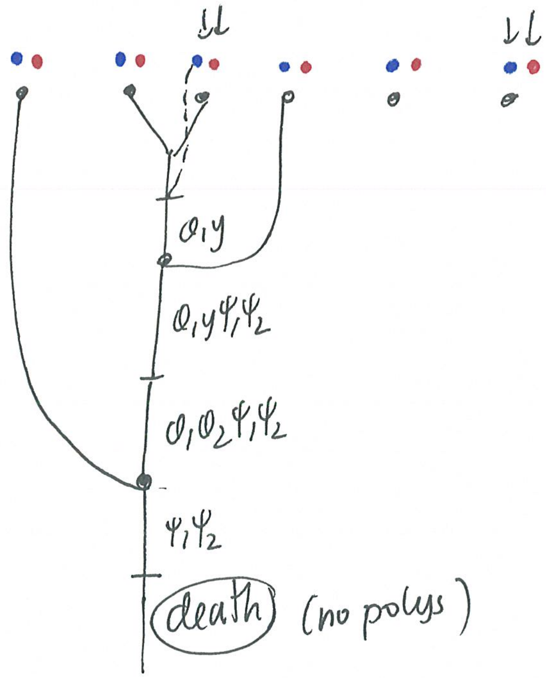



We continue (7.1), (7.2) to

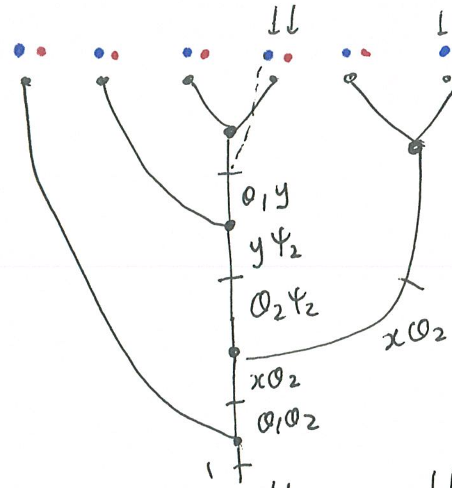


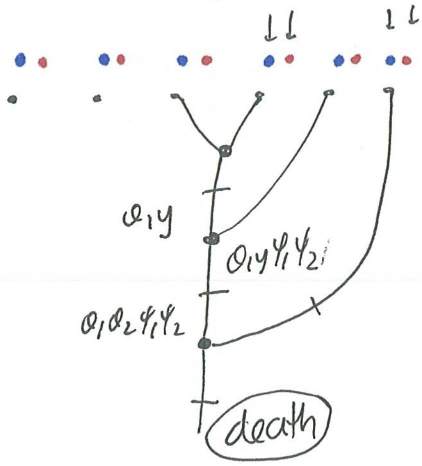

<u>Conclusion</u> No diagrams with (2,3) as first pair make a contribution

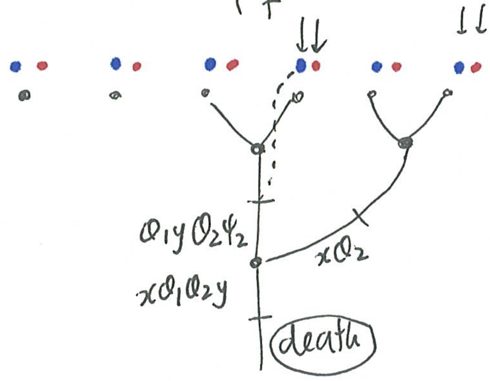
Now consider (3,4) as first pair, but not (2,3)

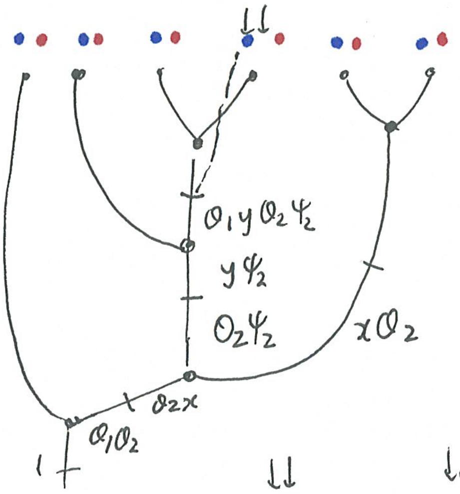



nonzero

A

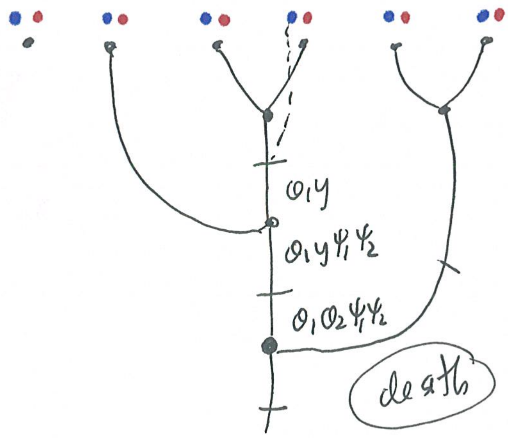


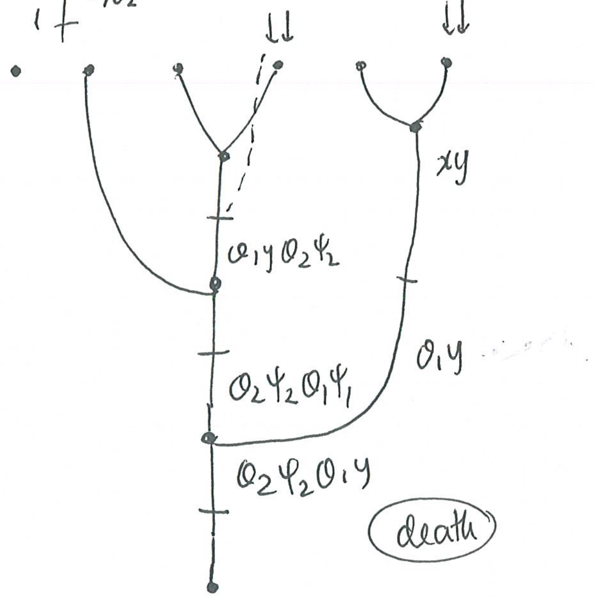



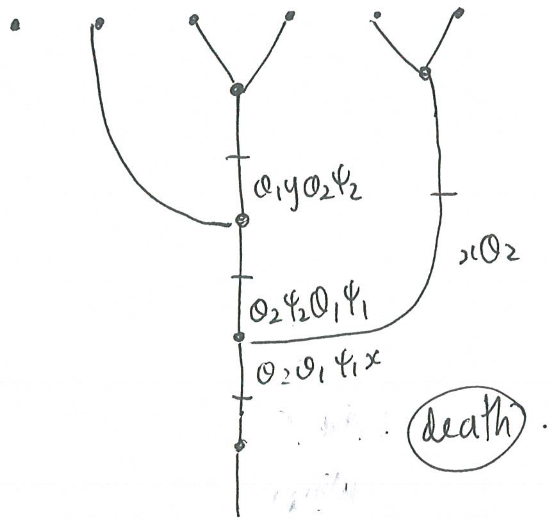

nonzero!

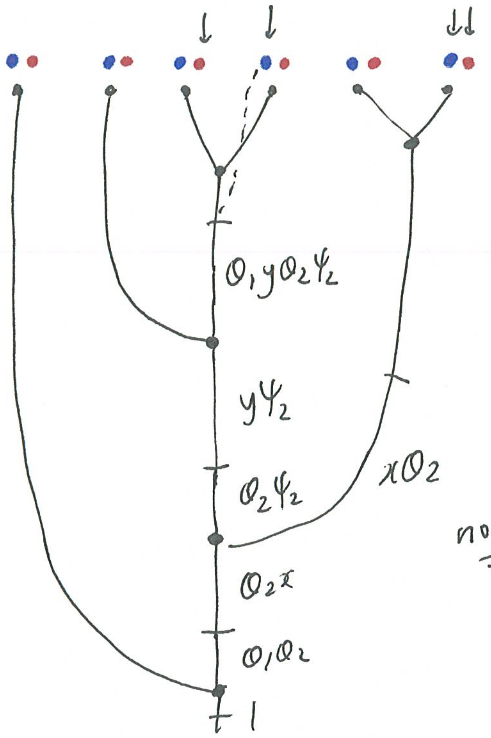
B



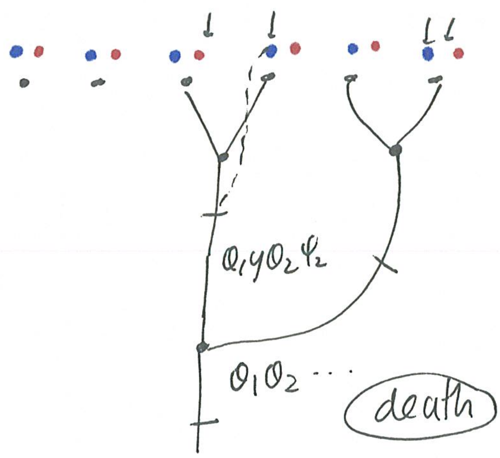

These are the only contributing diagrams where channel 4 has both primaries.

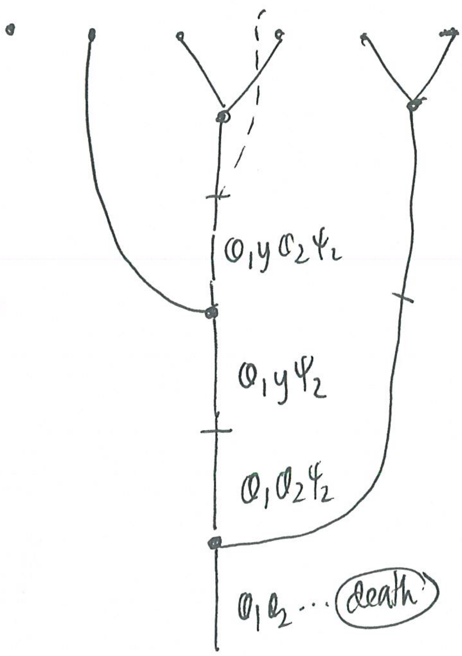





C

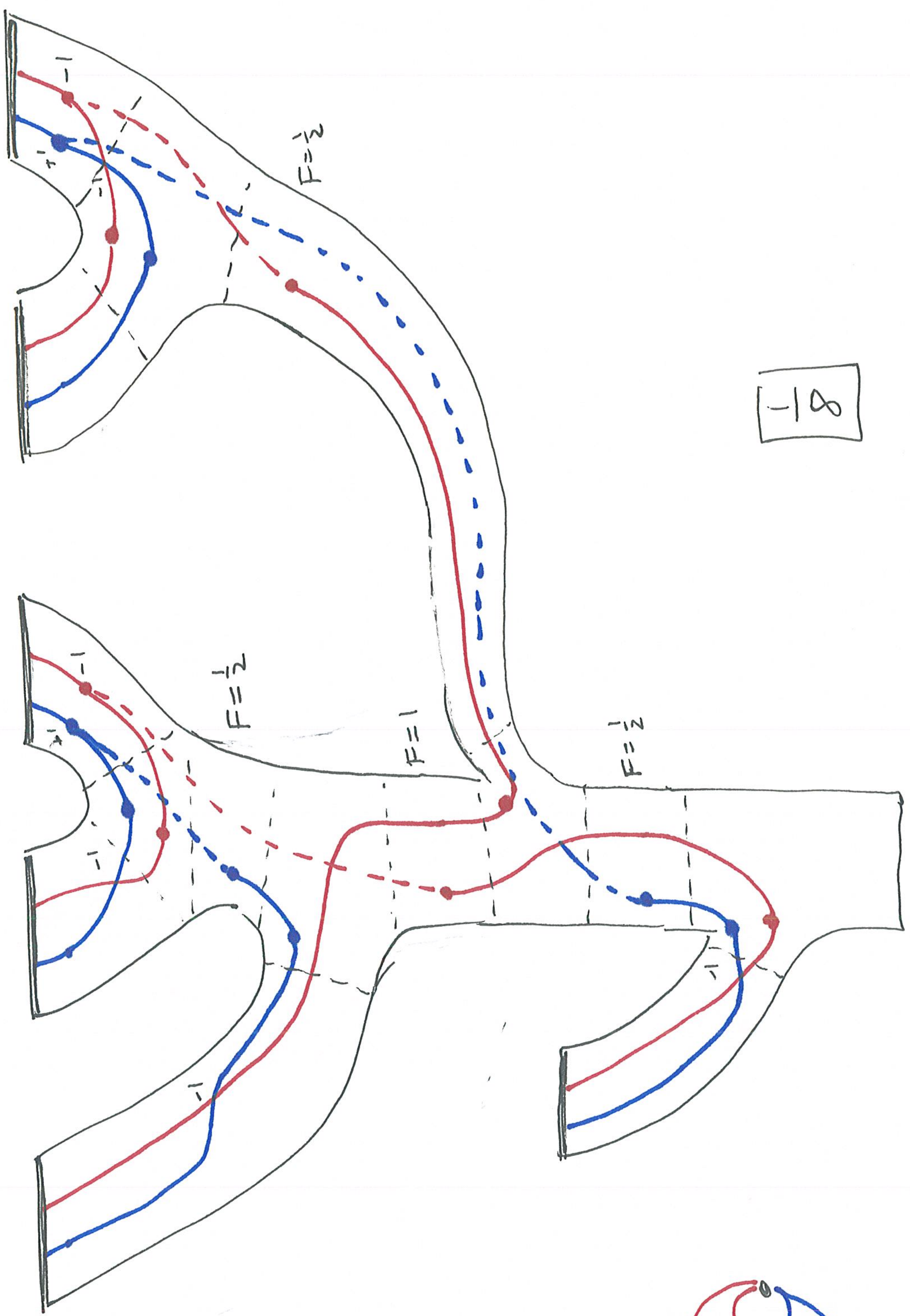
nonzero



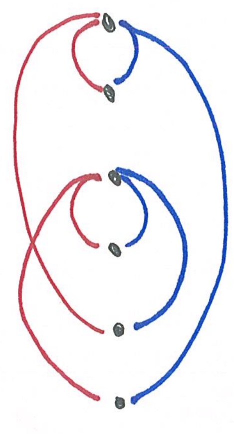




We have identified ~~four~~ six contributing diagrams. Three on p. 9, 10 and their blue/red switched versions.

To compute the amplitude for (A), we have



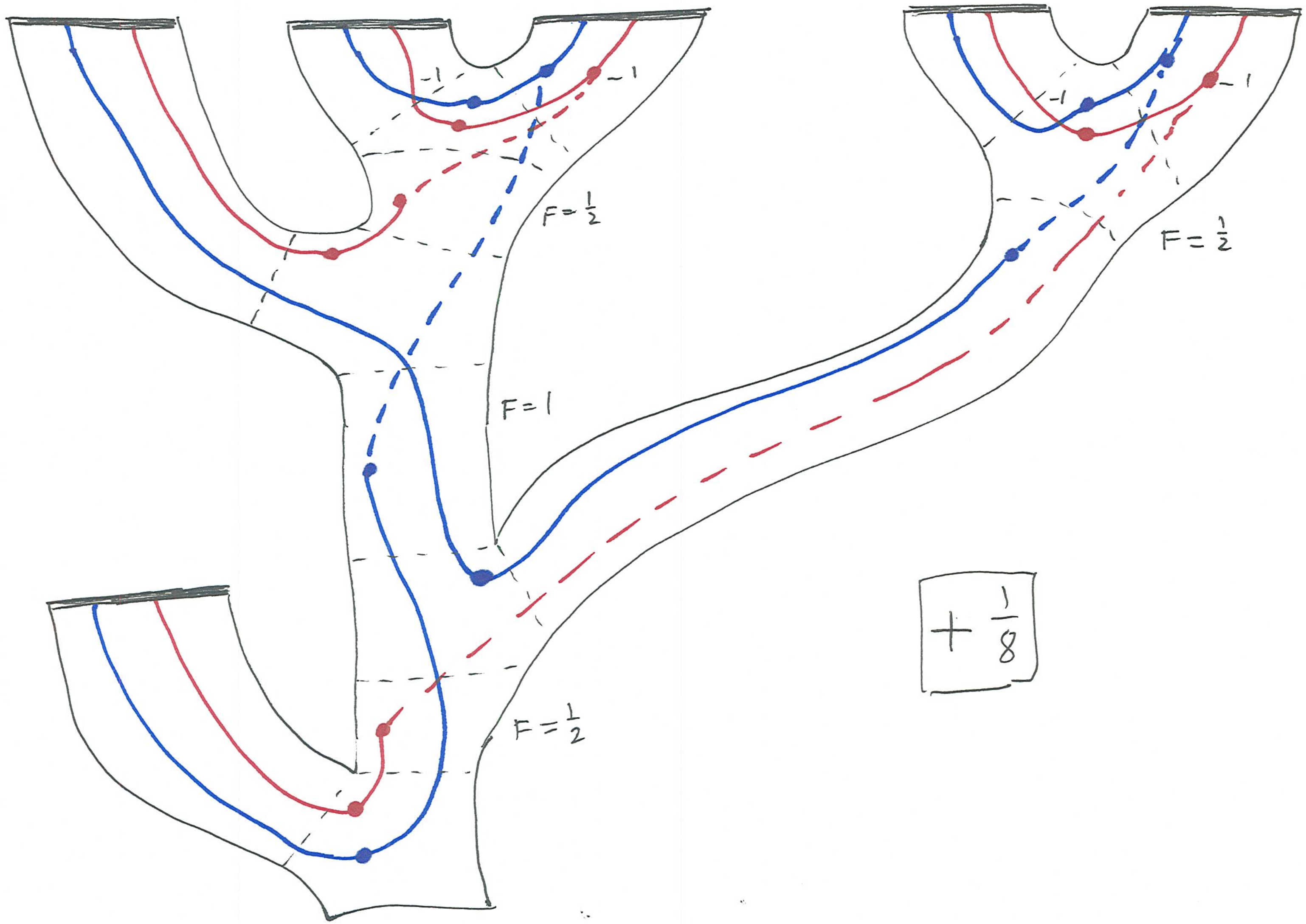

$\boxed{\frac{1}{8}}$



$(+1)(-1)(+1)(-1)(-1)(-1)(-1)(-1) = \boxed{+1}$

(A)

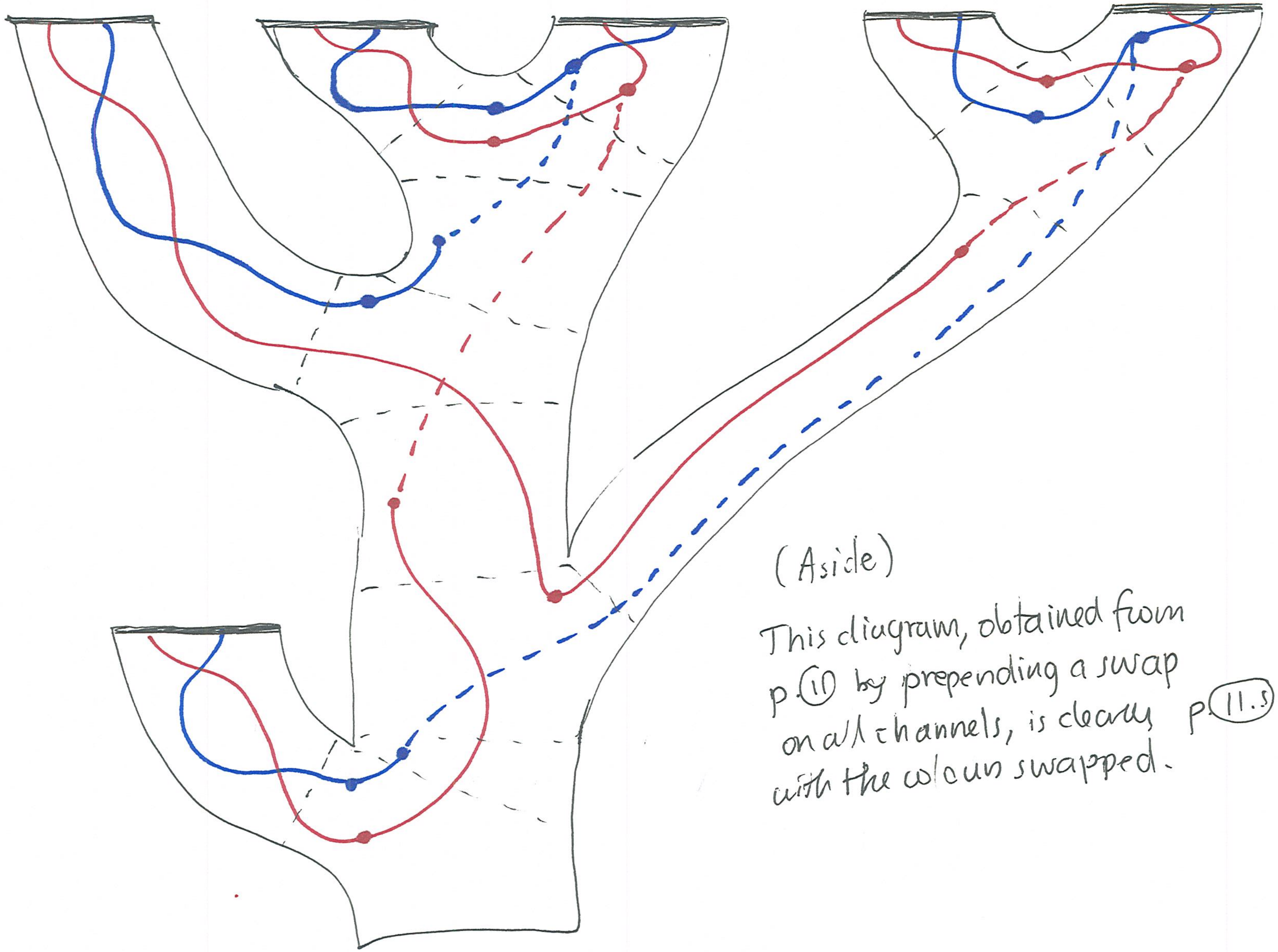
(A) with roles interchanged.




$\boxed{+\frac{1}{8}}$

ainfmf11

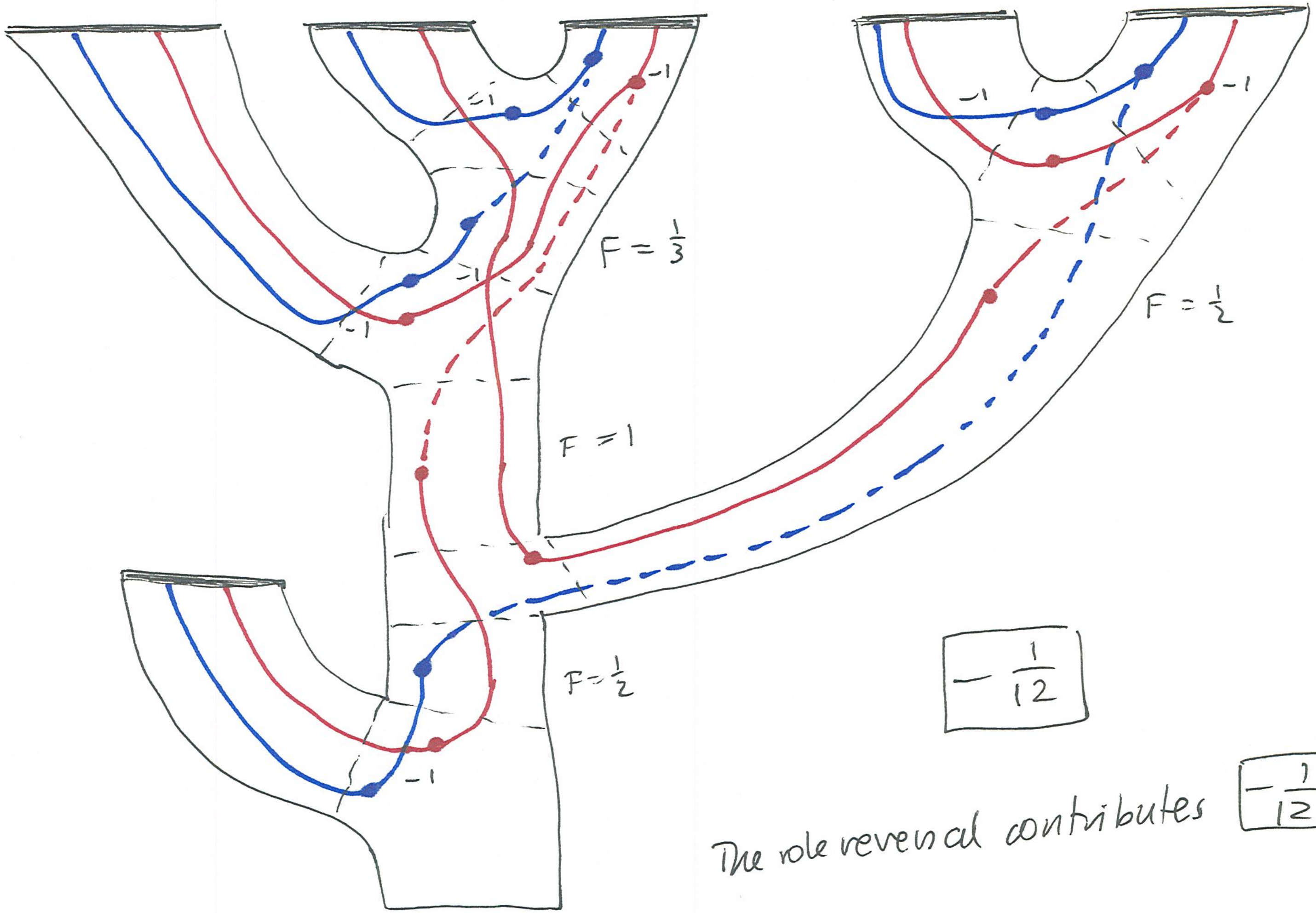
11.5



(Aside)

This diagram, obtained from p.11 by prepending a swap on all channels, is clearly p.11.5 with the colours swapped.

(B)

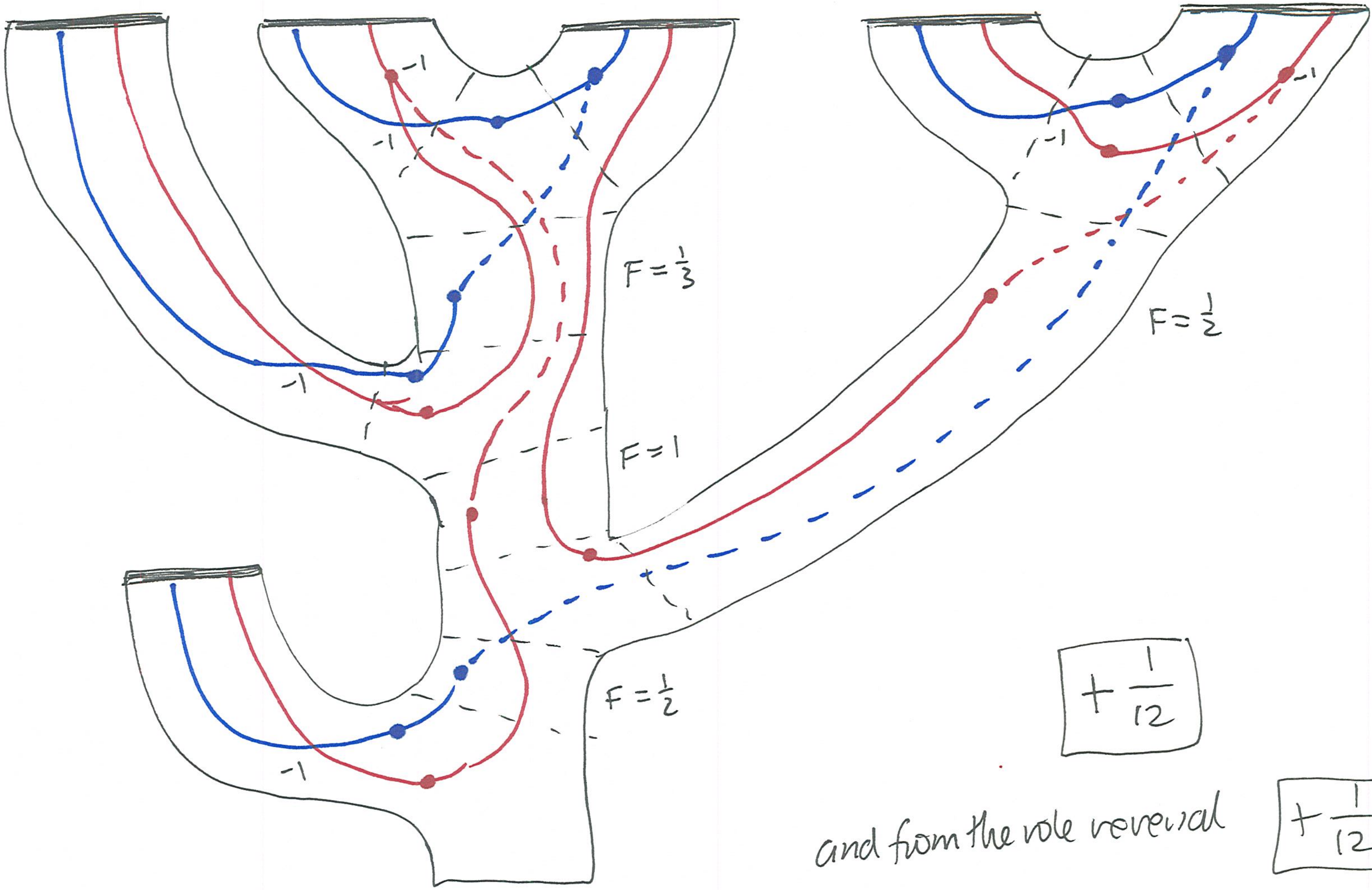



$$\boxed{-\frac{1}{12}}$$

The role reversal contributes $\boxed{-\frac{1}{12}}$ also.

$$(-1)(-1)(-1)(-1)(-1)(-1)(-1) = -1$$

(c)




$\boxed{+\frac{1}{12}}$

and from the role reversal $\boxed{+\frac{1}{12}}$

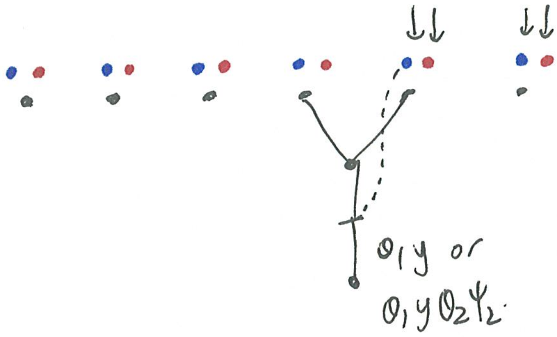
We can see why this cancels with
(B) by considering the $\Psi_2$'s in the 3,4-channel.

(13)

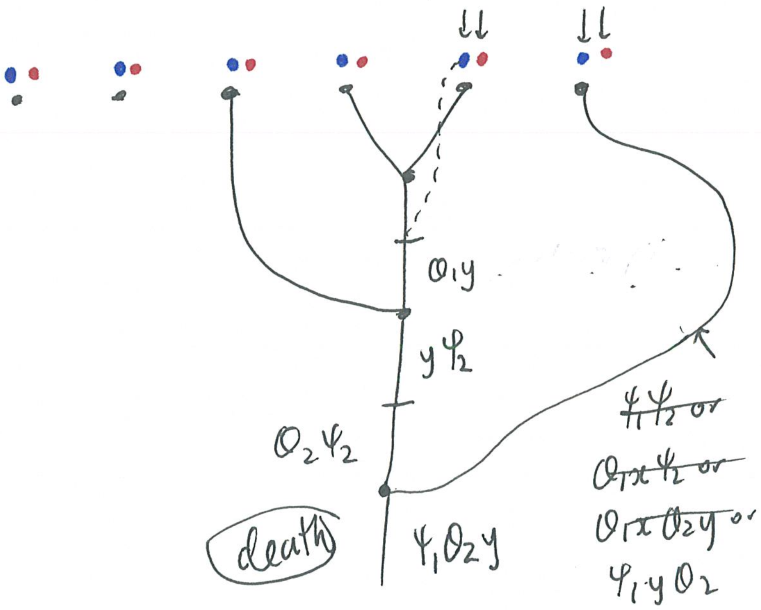
ainfmf11

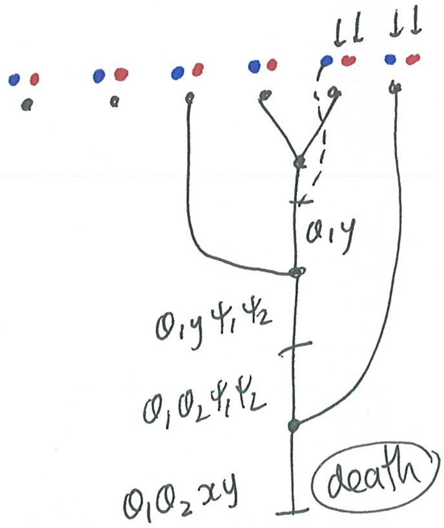
The remaining possibilities have neither (2,3) or (3,4) as first pair.

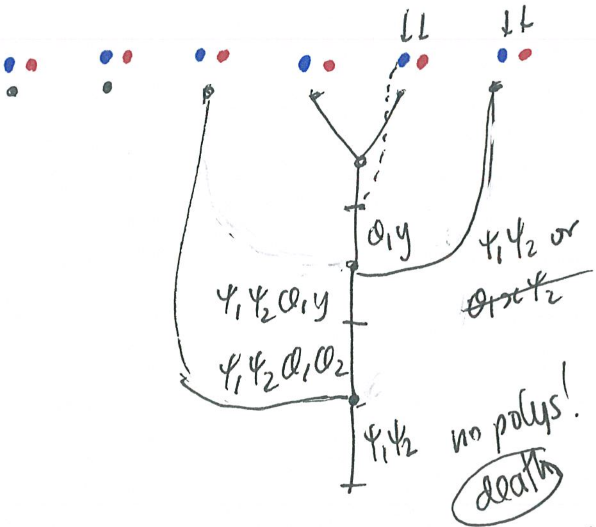
(4,5) is a first pair, and assume both primaries on 5th channel

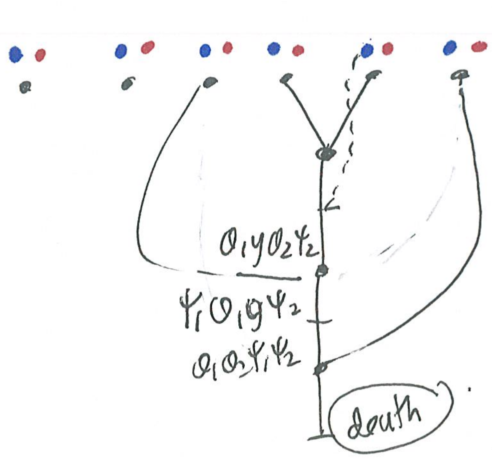



At this point the only options are to join in channel 3 or 6,

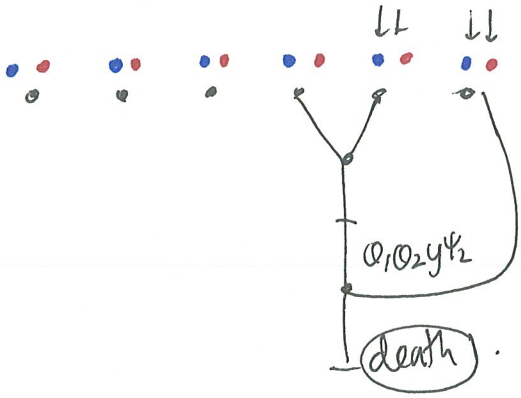


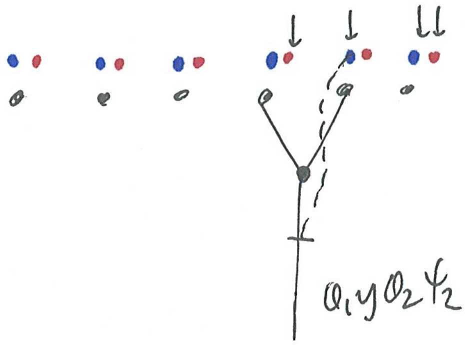




(of course $\psi_1, \psi_2$ can make them but they have no chance)



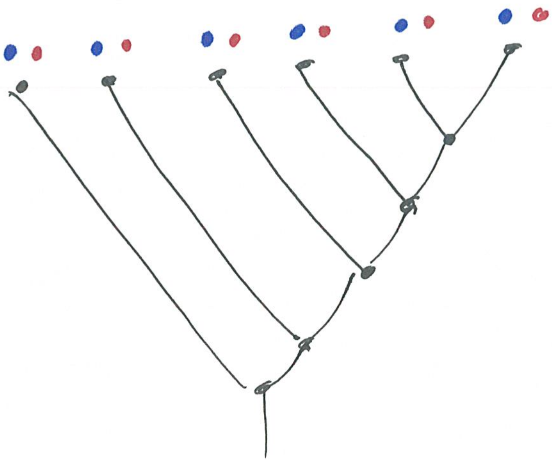

$\therefore$ $(4,5)$ cannot be a first pair if both primaries are on channel 5. Otherwise




$\leftarrow$ already treated.

<u>Conclusion</u> $(4,5)$ is not a first pair (in the case where neither $(2,3)$ or $(3,4)$ are.

What remains: neither $(2,3)$, $(3,4)$ or $(4,5)$ are first pairs but $(5,6)$ is



which we know vanishes by p. 5. So the only contributions are from $(3,4)$ as a first pair.

we conclude

$$b_6 ( \psi_1^* \psi_2^* \otimes \cdots \otimes \psi_1^* \psi_2^* )_{const} = 1/4$$

The only contribution is from A on p. 9 (and the red-blue swap, both contribute + 1/8)

NOTE There are probably sign corrections, but we will make these elsewhere.